

Time-dependent spherically symmetric covariant Galileons

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Abstract

We study spherically symmetric solutions of the cubic covariant Galileon model in curved space-time in presence of a matter source, in the test scalar field approximation. We show that a cosmological time evolution of the Galileon field gives rise to an *induced* matter-scalar coupling, due to the Galileon-graviton kinetic braiding, therefore the solution for the Galileon field is non trivial even if the bare matter-scalar coupling constant is set to zero. The local solution crucially depends on the asymptotic boundary conditions, and in particular, Minkowski and de Sitter asymptotics correspond to different branches of the solution. We study the stability of these solutions, namely, the well-posedness of the Cauchy problem and the positivity of energy for scalar and tensor perturbations, by diagonalizing the kinetic terms of the spin-2 and spin-0 degrees of freedom. In addition, we find that in presence of a cosmological time evolution of the Galileon field, its kinetic mixing with the graviton leads to a friction force, resulting to efficient damping of scalar perturbations within matter.

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I. INTRODUCTION

A scalar-tensor theory of gravity with higher-order derivative terms in the action, but with no more than second derivatives in the field equations, has first been found in Ref. [1]. Later, a subclass of the scalar part of this theory has been discussed in a different context in [2]. In the framework of the Dvali-Gabadadze-Poratti (DGP) brane model [3], the cubic scalar higher derivative action appears naturally in a specific decoupling limit. More recently, a generalization of the DGP decoupling limit action has been presented in [4], where this model was dubbed “Galileon”, because the considered action is invariant under the galilean transformation of the field (adding a constant to the field or a constant vector to its gradient). The original Galileon model has been later covariantized to include a dynamical metric [5], and then extended and generalized in several papers, see, e.g. [6, 7].

The fact that the field equations are no more than second order, in spite of the higher-order action, guarantees the absence of any Ostrogradsky ghost, i.e., an extra degree of freedom with negative kinetic energy. Another interesting feature of the Galileon model is that it generically possesses a Vainshtein screening mechanism. This last property can be understood as a particular case of “k-mouflage”, i.e., of kinetic screening [8]. The Vainshtein mechanism hides the propagating scalar degree of freedom, thus providing a way to pass local gravity tests, even if the scalar field is directly coupled to matter. The local effects of the Vainshtein screening of the Galileon has been studied in a number of works [9].

Usually, when studying local effects of the Galileon, the cosmological evolution of the scalar field is not taken into account, because of the smallness of the time derivative of the field compared to the spacial gradients. However, such a disregard of the Galileon’s time evolution may be misleading. For example, it was shown in Ref. [10] that in spite of the Vainshtein mechanism, which operates locally, this time evolution still poses severe constraints on many interesting scalar-tensor theories possessing a shift symmetry and a matter-scalar coupling.

In this work, we study the behavior of the covariant cubic Galileon model (the covariant version of the decoupling limit of the DGP model) in presence of a spherically symmetric matter source, and possible time evolution of the scalar field imposed by cosmology. We investigate in detail two distinct cases: (i) a spherically symmetric matter source is embedded in asymptotically Minkowski spacetime, and the Galileon field does not depend on time; (ii) asymptotically de Sitter Universe with time-dependent Galileon field. To realize the latter regime, we adopt the cosmological solution of the Kinetically Gravity Braiding model [11], for which the acceleration of the Universe is driven by the time evolving Galileon field itself. We find non-homogeneous solutions in the test scalar field approximation, and analyze the stability of perturbations on top of these solutions.

The paper is organized as follows. In Sec. II, we introduce the action and field equations of the cubic covariant Galileon. In Sec. III, we derive a solution for a scalar field having a cosmological time evolution, in an arbitrary spherically symmetric static spacetime, in the test field approximation. Perturbations are investigated in Sec. IV, where we diagonalize the kinetic matrix, demixing the spin-0 and spin-2 propagating modes. The solution in asymptotically Minkowski spacetime, in presence of a matter source with no time evolution of the scalar field, is studied in Sec. V, where we find the effective metric for the helicity-0 propagating mode, and the conditions for stability of the solution. In Sec. VI, we perform a similar analysis for asymptotically de Sitter spacetime, in presence of a cosmological evolution of the scalar field. Scalar perturbations inside a matter source are investigated in

Sec. VII. Besides the analyses of the Cauchy problem and the positivity of energy, we also exhibit a large friction term, which is caused by the cosmologically-imposed time evolution of the Galileon within matter. Our conclusions are presented in Sec. VIII.

II. ACTION AND FIELD EQUATION

We consider the quadratic plus cubic Galileon action in curved spacetime,

$$S = 2M_P^2 \int d^4x \sqrt{-g} \left\{ \frac{R}{4} - \frac{k_2}{2} (\partial_\mu \varphi)^2 - \frac{k_3}{2M^2} \square \varphi (\partial_\mu \varphi)^2 \right\} + S_{\text{matter}}[\psi_{\text{matter}}; \tilde{g}_{\mu\nu}], \quad (1)$$

where the scalar field φ is chosen to be dimensionless, where the reduced Planck mass¹ $M_P \equiv (8\pi G)^{-1/2}$ should not be confused with the mass scale M entering the Galileon's cubic kinetic term, and where g and R denote respectively the determinant and the scalar curvature of the metric $g_{\mu\nu}$ (of signature $-+++$) used to define covariant derivatives and to contract indices. All matter fields, globally denoted as ψ_{matter} , are assumed to be minimally coupled to a physical (“Jordan”) metric $\tilde{g}_{\mu\nu} \equiv e^{2\alpha\varphi} g_{\mu\nu}$, where α denotes a dimensionless matter-scalar coupling constant. The constants k_2 and k_3 , dimensionless too, could *a priori* be reabsorbed in the definitions of the scalar field φ and of the mass scale M (while changing simultaneously the value of α : $\varphi^{\text{new}} = |k_2|^{1/2} \varphi$, $M^{\text{new}} = M|k_2|^{3/4}/|k_3|^{1/2}$, $\alpha^{\text{new}} = \alpha/|k_2|^{1/2}$). In other words, the model (1) actually depends on only two independent parameters, for instance M and α . However, the redundant factors k_2 and k_3 will be useful in the following to keep track of the origin of the different terms, and above all to change easily the signs of these two kinetic terms. Their numerical factors are chosen to simplify the following results involving k_3 , and so that $k_2 = 1$, $k_3 = 0$ defines a canonically normalized positive-energy spin-0 degree of freedom. Another interest of these coefficients is that any normalization convention can easily be recovered, for instance $k_2 = \frac{1}{2}$ corresponding to the relative weight of the Einstein-Hilbert action and the scalar-field standard kinetic term in many cosmological papers.

When $k_3 = 0$, i.e., in standard scalar-tensor theories of gravity [12–14], $g_{\mu\nu}$ is called the “Einstein metric” and its fluctuations describe a spin-2 degree of freedom. As we will see in Sec. IV, this is no longer the case when $k_3 \neq 0$ because of the metric-scalar derivative coupling entering the Galileon's cubic kinetic term. The metric $g_{\mu\nu}$ is thus neither the Jordan ($\tilde{g}_{\mu\nu}$) nor the Einstein one, and it should just be considered as the variable we choose to write the field equations as simply as possible.

We shall study a *test* scalar field in the background metric generated by a spherical body, assuming that its backreaction on this metric is negligible, which is always the case for a small enough matter-scalar coupling constant². We shall also check *a posteriori* in which conditions this backreaction is indeed negligible. It will thus suffice to focus first on the scalar field equation, which can be written as

$$\nabla_\mu J^\mu = -\alpha T, \quad (2)$$

¹ Throughout this paper, we choose natural units such that $\hbar = c = 1$.

² Anticipating on definition (10) below, we are actually talking here about the *effective* matter-scalar coupling constant α_{eff} , which takes into account the induced coupling in presence of a cosmological time evolution of the scalar field.

where $J^\mu \equiv -(1/\sqrt{-g})\delta S/\delta\partial_\mu\varphi$ is the scalar field's current, and T denotes the trace of $T^{\mu\nu} \equiv (2/\sqrt{-g})(\delta S_{\text{matter}}/\delta g_{\mu\nu})$, related to the physical (Jordan-frame) matter energy-momentum tensor $\tilde{T}^{\mu\nu} \equiv (2/\sqrt{-\tilde{g}})(\delta S_{\text{matter}}/\delta\tilde{g}_{\mu\nu})$ by $T^{\mu\nu} = e^{6\alpha\varphi}\tilde{T}^{\mu\nu} \Rightarrow T = e^{4\alpha\varphi}\tilde{T}$. The current J^μ reads explicitly

$$\frac{1}{M_P^2}J^\mu = 2k_2\partial^\mu\varphi + 2\frac{k_3}{M^2}\Box\varphi\partial^\mu\varphi - \frac{k_3}{M^2}\nabla^\mu((\partial_\lambda\varphi)^2), \quad (3)$$

and Eq. (2) takes thus the full form

$$k_2\Box\varphi + \frac{k_3}{M^2}\{(\Box\varphi)^2 - (\nabla_\mu\partial_\nu\varphi)^2 - R^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\} = -\frac{\alpha}{2M_P^2}T, \quad (4)$$

where the Ricci tensor $R^{\mu\nu}$ enters because of a difference of third-order covariant derivatives of the scalar field: $[\nabla_\mu\nabla_\nu - \nabla_\nu\nabla_\mu]\nabla^\nu\varphi = -R^\nu_\mu\partial_\nu\varphi$. Note that this curvature tensor involves second derivatives of the metric, therefore Eq. (4) actually mixes spin-0 and spin-2 degrees of freedom. We will diagonalize them when studying perturbations in Sec. IV, but to derive the spherically-symmetric solution for a test scalar field, it suffices to replace this Ricci tensor by its vacuum value $R^{\mu\nu} = 0$ outside the central massive body.

III. BACKGROUND SOLUTION

A static and spherically symmetric background metric reads in Schwarzschild coordinates

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2d\Omega^2, \quad (5)$$

where $\lambda(r)$ and $\nu(r)$ are two functions of the radial coordinate. The exterior (vacuum) solution is well known to read $e^\nu = e^{-\lambda} = 1 - r_S/r$ in an asymptotically flat spacetime, where $r_S \equiv 2Gm$ denotes the Schwarzschild radius of the central body of mass m , but we will also consider the Schwarzschild-de Sitter solution in Sec. VI below. We look for a test scalar field solution of Eq. (2) or (4), assuming that the cosmological evolution imposes a linear time dependence, i.e., that $\ddot{\varphi} = 0$ (where a dot denotes a time derivative). As shown in [11], such a linear time-evolution is a cosmological attractor. Let us look for a solution of the form

$$\varphi = \phi(r) + \dot{\varphi}_c t + \varphi_0, \quad (6)$$

where $\dot{\varphi}_c$ and φ_0 are assumed to be constants. The ansatz (6) has been used in a similar context for the study of the Galileon accretion [15] and the effects of cosmologically evolving scalar field on the variation of the Newton constant [10]. Actually, since $\varphi' = \phi'$ (where a prime denotes a radial derivative), the notation ϕ will not be useful in the following, and we can thus write Eq. (2) in terms of φ' . With the ansatz (6), this field equation reduces in vacuum to a mere $\partial_r[r^2e^{(\lambda+\nu)/2}J^r] = 0$, and it can be integrated once as $e^{(\lambda+\nu)/2}J^r = \alpha M_P^2 r_S/r^2$, the constant of integration being imposed by the same Eq. (2) within matter.³

³ Actually, this constant of integration is slightly modified by self-gravity effects within the body, because the scalar field is sourced by the *trace* of the matter energy-momentum tensor, with a different pressure dependence than in Einstein's equations. We should thus write rigorously, like in Brans-Dicke theory [13, 14], $e^{(\lambda+\nu)/2}J^r = \alpha(1-2s)M_P^2 r_S/r^2$, where $s \equiv -\partial \ln m / \partial \ln G \sim |E_{\text{grav}}|/m \sim r_S/r_{\text{body}}$.

In terms of φ' , this reads

$$\frac{2k_3}{M^2 r} \left(1 + \frac{\nu' r}{4}\right) e^{-\lambda} \varphi'^2 + k_2 \varphi' - \frac{e^{(\lambda-\nu)/2}}{2} \left(\frac{\alpha r_S}{r^2} + \frac{k_3}{M^2} \dot{\varphi}_c^2 \frac{\nu'}{e^{(\lambda+\nu)/2}} \right) = 0, \quad (7)$$

whose two solutions are

$$\varphi' = -\frac{k_2 M^2 e^\lambda r}{4k_3 \left(1 + \frac{\nu' r}{4}\right)} \left[1 \pm \sqrt{1 + \frac{4k_3 r_S}{k_2^2 M^2 r^3 e^{(\lambda+\nu)/2}} \left(1 + \frac{\nu' r}{4}\right) \left(\alpha + \frac{k_3}{M^2} \dot{\varphi}_c^2 \frac{\nu' r^2}{r_S e^{(\lambda+\nu)/2}} \right)} \right]. \quad (8)$$

Note that for a test scalar field, whose backreaction on the metric is negligible, these expressions (8) are exact, i.e., correct to any post-Newtonian order (while taking into account the slight numerical change of $\alpha \rightarrow \alpha(1 - 2s)$ due to self-gravity, as underlined in footnote 3). Of course, for the exterior Schwarzschild solution, they can be simplified because $e^{\lambda+\nu} = 1$. Moreover, for $r \gg r_S$, one may expand λ and ν in powers of r_S/r , and get

$$\varphi' = -\frac{k_2 M^2 r}{4k_3} \left(1 \pm \sqrt{1 + \frac{4k_3 r_S}{k_2^2 M^2 r^3} \alpha_{\text{eff}}} \right) \times \left[1 + \mathcal{O}\left(\frac{r_S}{r}\right) \right], \quad (9)$$

where

$$\alpha_{\text{eff}} \equiv \alpha + \frac{k_3 \dot{\varphi}_c^2}{M^2} \quad (10)$$

is an effective matter-scalar coupling constant, modifying the bare one, α , because of the self-interaction of the scalar field with its own cosmologically-imposed time evolution $\dot{\varphi}_c$. Let us underline that even if one assumes $\alpha = 0$ in action (1), i.e., *a priori* no matter-scalar interaction, the cosmological time evolution of the scalar field does generate a nonvanishing coupling to matter. This is due to the quadratic terms entering the current (3), since the Christoffel symbols do depend on the background metric and thereby on the mass of the central body.

We will discuss in Secs. V and VI which sign should be chosen in solution (8), by studying the large-distance behavior of the scalar field. We will see in particular that it does not reduce to (9) at cosmologically large distances if spacetime is not assumed to be asymptotically flat (see Eqs. (31) to (33)). At present, let us just note that the square root entering (9) involves a contribution $\propto 1/r^3$, dominating at small enough distances (still assumed to be much larger than the Schwarzschild radius r_S). We define the “Vainshtein radius” as

$$r_V \equiv \left(\frac{4k_3 r_S}{M^2 k_2^2} \alpha_{\text{eff}} \right)^{1/3}. \quad (11)$$

It is positive only if $k_3 \alpha_{\text{eff}} > 0$, which is always the case when the bare $\alpha = 0$, but depends on the model when $\alpha \neq 0$. When $r_V < 0$, then Eqs. (8) or (9) simply do not give an acceptable solution for the scalar field at radii $r < |r_V|$. This means that the ansatz (6) cannot be correct at such small distances, and the actual solution must mix its time and radial dependences in a more subtle way. When r_V is positive, on the other hand, Eq. (8) or (9) is a correct solution. In the range $r_S \ll r \ll r_V$, the scalar field then takes the form

$$\varphi' = \mp \frac{\text{sign}(k_2 k_3)}{2} \sqrt{\frac{r_S}{r}} \sqrt{\frac{M^2 \alpha_{\text{eff}}}{k_3}} \left[1 + \mathcal{O}\left(\frac{r_S}{r}\right) + \mathcal{O}\left(\frac{r^3}{r_V^3}\right) \right] \quad (12a)$$

$$= \mp \frac{k_2 M^2}{4k_3} \sqrt{\frac{r_V^3}{r}} \left[1 + \mathcal{O}\left(\frac{r_S}{r}\right) + \mathcal{O}\left(\frac{r^3}{r_V^3}\right) \right]. \quad (12b)$$

Note in passing that the product $k_3\varphi'$, which will play an important role in the following sections, has a sign which does not depend on k_3 (provided this Vainshtein regime exists, i.e., notably that $k_3\alpha_{\text{eff}} > 0$), but which does depend on the sign of k_2 , on the contrary.

It should be underlined that this regime $r_S \ll r \ll r_V$ can exist even when $\alpha = 0$, provided $|2k_3\dot{\varphi}_c/k_2M^2| \gg r_S$. Therefore, although one could have naively deduced from (8)-(9) that φ' either vanishes or is proportional to r (depending on the sign to be taken in this solution), Eq. (12) actually proves it is proportional to $r^{-1/2}$. The homogeneous scalar field assumed in Ref. [11] thus cannot be used in the vicinity of a massive body, when there is a nonzero cosmological time evolution $\dot{\varphi}_c \neq 0$. It would remain possible to assume that $\alpha_{\text{eff}} = 0$, but this would need to fine-tune the cosmologically-driven time derivative $\dot{\varphi}_c$ to the precise value $(-\alpha/k_3)^{1/2}M$ involving the constants parameters of action (1) (while also assuming that $\alpha k_3 < 0$).

IV. PERTURBATIONS

Let us now study the perturbations of both the metric and the scalar field around the background solution defined by Eqs. (5) and (8). We write $g_{\mu\nu}^{\text{full}} = g_{\mu\nu} + h_{\mu\nu}$ and $\varphi^{\text{full}} = \varphi + \pi$, where $g_{\mu\nu}$ and φ denote the background fields, and $h_{\mu\nu}$ and π their perturbations. Our aim is to identify the pure helicity-2 and 0 degrees of freedom, to analyze in which conditions they both carry positive energy, so that any ghost instability is avoided. We expand action (1) to second order in the perturbations, and keep only the kinetic terms, which involve at least two derivatives of these fields. Of course, terms of the form $f(\text{background})h\nabla\nabla\pi$ can be integrated by parts, and contribute thus to the kinetic terms as $-f(\text{background})\nabla h\nabla\pi$. Because of the cubic Galileon interaction in action (1), one actually also gets a term $\propto 2\partial_\mu\varphi\partial^\mu\pi\Box\pi$, involving *three* derivatives of the scalar perturbation π , but this can be rewritten as $\Box\varphi(\partial_\mu\pi)^2 - 2\nabla_\mu\partial_\nu\varphi\partial^\mu\pi\partial^\nu\pi$ by partial integration, as expected because the Galileon field equations (perturbed or not) are known to involve at most second derivatives. The final sum of all kinetic terms for the perturbations $h_{\mu\nu}$ and π reads

$$\begin{aligned} \frac{1}{2M_P^2} \frac{\mathcal{L}_2^{\text{kinetic}}}{\sqrt{-g}} = & -\frac{1}{8}\nabla_\mu h_{\alpha\beta} P^{\alpha\beta\gamma\delta} \nabla^\mu h_{\gamma\delta} + \frac{1}{8} \left(h_{\nu;\lambda}^\lambda - \frac{1}{2} h_{,\nu} \right)^2 - \frac{k_2}{2} (\partial_\mu\pi)^2 \\ & - \frac{k_3}{2M^2} \left[2\Box\varphi (\partial_\mu\pi)^2 - 2\nabla_\mu\partial_\nu\varphi \partial^\mu\pi\partial^\nu\pi + \partial_\mu\varphi\partial_\nu\varphi \partial_\lambda\pi\nabla^\lambda h^{\mu\nu} \right. \\ & \left. - 2\partial^\mu\varphi\partial^\nu\varphi \partial_\mu\pi \left(h_{\nu;\lambda}^\lambda - \frac{1}{2} h_{,\nu} \right) \right], \end{aligned} \quad (13)$$

where $P^{\alpha\beta\gamma\delta} \equiv \frac{1}{2}g^{\alpha\gamma}g^{\beta\delta} - \frac{1}{4}g^{\alpha\beta}g^{\gamma\delta}$, the indices of $h_{\mu\nu}$ are raised with the background metric $g^{\rho\sigma}$, and $h \equiv h_\lambda^\lambda$ is the trace of $h_{\mu\nu}$. The presence of cross terms $f(\text{background})\nabla\pi\nabla h$ illustrates that the helicity-2 and 0 degrees of freedom are mixed. But a simple change of variables actually suffices to diagonalize these kinetic terms. We define

$$\tilde{h}_{\mu\nu} \equiv h_{\mu\nu} + \frac{4k_3}{M^2} \left[\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g_{\mu\nu}(\partial_\lambda\varphi)^2 \right] \pi, \quad (14)$$

and Eq. (13) then takes the form

$$\frac{1}{2M_P^2} \frac{\mathcal{L}_2^{\text{kinetic}}}{\sqrt{-g}} = -\frac{1}{8}\nabla_\mu \tilde{h}_{\alpha\beta} P^{\alpha\beta\gamma\delta} \nabla^\mu \tilde{h}_{\gamma\delta} + \frac{1}{8} \left(\tilde{h}_{\nu;\lambda}^\lambda - \frac{1}{2} \tilde{h}_{,\nu} \right)^2 - \frac{1}{2} \mathcal{G}^{\mu\nu} \partial_\mu\pi\partial_\nu\pi, \quad (15)$$

where

$$\mathcal{G}^{\mu\nu} \equiv g^{\mu\nu} \left[k_2 + \frac{2k_3}{M^2} \square \varphi - \frac{k_3^2}{M^4} (\partial_\lambda \varphi)^4 \right] - \frac{2k_3}{M^2} \nabla^\mu \partial^\nu \varphi + 4 \frac{k_3^2}{M^4} (\partial_\lambda \varphi)^2 \partial^\mu \varphi \partial^\nu \varphi, \quad (16)$$

proving that $\tilde{h}_{\mu\nu}$ describes a pure helicity-2 field propagating in the curved background of $g_{\mu\nu}$, and that the pure helicity-0 field π propagates in the effective metric $\mathcal{G}^{\mu\nu}$. Our full diagonalization recovers thus this effective metric first derived in [11], from a triangulation of the kinetic terms (using the clever trick of replacing the Ricci tensor entering Eq. (4) by its source, obtained from the Einstein equations). The $\mathcal{O}(k_3^2)$ terms are the only subtleties introduced by this diagonalization, but we will see in the following that they are subdominant in natural situations. The crucial information brought by Eq. (15) is that the spin-2 graviton is never a ghost, and that the effective metric (16) should be of signature $-+++$ for the scalar perturbation π to carry positive energy and have a well-posed Cauchy problem. We shall perform this analysis in Secs. V and VI, for two different asymptotic conditions.

Before entering this discussion, let us underline that our diagonalization (15) also allows us to exhibit the induced matter-scalar coupling already noticed in Eq. (10) for the background solution. We can show now that the scalar perturbation π is also directly coupled to matter, even when the bare coupling constant α vanishes. Indeed, since matter is assumed to be minimally coupled to $\tilde{g}_{\mu\nu}^{\text{full}} = \exp(2\alpha\varphi^{\text{full}})g_{\mu\nu}^{\text{full}}$ (where “full” means as before the background fields plus their perturbations), the action of a point particle reads

$$\begin{aligned} S_{\text{matter}} &= - \int mc (\tilde{g}_{\mu\nu}^{\text{full}} dx^\mu dx^\nu)^{1/2} \\ &= - \int mc e^{\alpha\varphi} [1 + \alpha\pi + \mathcal{O}(\pi^2)] (g_{\mu\nu} dx^\mu dx^\nu)^{1/2} \left[1 - \frac{1}{2} h_{\rho\sigma} u^\rho u^\sigma + \mathcal{O}(h^2) \right], \end{aligned} \quad (17)$$

where $u^\lambda \equiv dx^\lambda / (g_{\mu\nu} dx^\mu dx^\nu)^{1/2}$ denotes the unit 4-velocity of the particle (with respect to the background metric $g_{\mu\nu}$). Although there does not seem to exist any linear coupling to π when $\alpha = 0$, we actually know that $h_{\mu\nu}$ describes a mixing of spin-2 and spin-0 degrees of freedom. If we replace it in terms of π and the actual spin-2 excitation $\tilde{h}_{\mu\nu}$, Eq. (14), we thus get

$$\begin{aligned} S_{\text{matter}} &= - \int mc (\tilde{g}_{\mu\nu} dx^\mu dx^\nu)^{1/2} \left[1 + \left\{ \alpha + \frac{2k_3}{M^2} (u^\lambda \partial_\lambda \varphi)^2 + \frac{k_3}{M^2} (\partial_\lambda \varphi)^2 \right\} \pi \right. \\ &\quad \left. - \frac{1}{2} \tilde{h}_{\rho\sigma} u^\rho u^\sigma + \mathcal{O}((\tilde{h}_{\rho\sigma}, \pi)^2) \right]. \end{aligned} \quad (18)$$

The quantity within curly brackets plays the role of an effective linear coupling constant of matter to the scalar perturbation π , and it can be nonzero even if $\alpha = 0$. It should *a priori* not be confused with α_{eff} , defined in Eq. (10), which described the effective coupling of the *background* scalar field φ to the matter source. However, at lowest post-Newtonian order, they actually coincide when they are constant, notably in a background such that $\dot{\varphi}_c \neq 0$ but $\partial_r \varphi = 0$. Indeed, we have $-(\partial_\lambda \varphi)^2 = e^{-\nu} \dot{\varphi}_c^2 = (u^\lambda \partial_\lambda \varphi)^2 + \mathcal{O}(v)$, where v is the particle’s velocity, so that the quantity within curly brackets in (18) reduces to (10).

V. ASYMPTOTIC MINKOWSKI SPACETIME

We consider in this section that spacetime is asymptotically Minkowskian, i.e., we impose that the spherically symmetric metric (5) is given by the Schwarzschild solution $e^\nu = e^{-\lambda} =$

$1 - r_S/r$, and we consistently assume that the background scalar field has no cosmological time evolution, $\dot{\varphi}_c = 0$. Then the upper sign of solution (8), (9) or (12) must be discarded, otherwise the scalar field would diverge at spatial infinity (as well as its derivative and its energy-momentum tensor). With the lower sign, Eq. (9) gives for $r \rightarrow \infty$ the standard behavior of a Brans-Dicke scalar field,

$$\varphi' = \frac{\alpha r_S}{2k_2 r^2} \left[1 + \mathcal{O}\left(\frac{r_S}{r}\right) + \mathcal{O}\left(\frac{r_V^3}{r^3}\right) \right] \quad (19a)$$

$$\Rightarrow \varphi = \varphi_0 - \frac{\alpha r_S}{2k_2 r} \left[1 + \mathcal{O}\left(\frac{r_S}{r}\right) + \mathcal{O}\left(\frac{r_V^3}{r^3}\right) \right]. \quad (19b)$$

In this large-distance regime, the perturbations of the scalar field also behave as in Brans-Dicke theory, i.e., they propagate in an effective metric (16) dominated by its standard $k_2 g^{\mu\nu}$ contribution. In conclusion, k_2 *must* be positive for the scalar degree of freedom to carry positive energy in the asymptotic $r \rightarrow \infty$ region.

On the other hand, in the Vainshtein regime $r \ll r_V$ (which exists only if $k_3\alpha > 0$), the background scalar field takes the form (12) with its lower sign, and the effective metric (16), in which scalar perturbations propagate, is dominated by its k_3 contributions. Indeed, we have $(2k_3/M^2)\square\varphi \approx \frac{3}{4}k_2(r_V/r)^{3/2} \gg k_2 > 0$. Similarly, the $-(2k_3/M^2)\nabla^\mu\partial^\nu\varphi$ contribution to $\mathcal{G}^{\mu\nu}$ gives at lowest post-Newtonian order $\frac{1}{4}k_2(r_V/r)^{3/2}$ for the \mathcal{G}^{rr} component, minus twice this expression (multiplied by $g^{\theta\theta}$ and $g^{\phi\phi}$) for the angular $\mathcal{G}^{\theta\theta}$ and $\mathcal{G}^{\phi\phi}$ components, and again this expression but multiplied by a negligible factor r_S/r for the \mathcal{G}^{00} component. Finally, if we assume that α^2/k_2 is at most of order one ($\alpha^2/k_2 \ll 1$ still being allowed), in order to avoid a too large matter-scalar coupling in the asymptotic $r \rightarrow \infty$ region, then the assumption $r_S \ll r \ll r_V$ implies $(k_3\varphi'^2/M^2)^2 \approx (\alpha r_S/4r)^2 \ll k_2 \ll (2k_3/M^2)\square\varphi$, therefore the $\mathcal{O}(k_3^2)$ terms entering the effective metric (16) are fully negligible: They are of second post-Newtonian order beyond the Vainshtein effect. In conclusion, the effective metric (16) reads at lowest order

$$\mathcal{G}_{\text{Vainshtein}}^{\mu\nu} \approx \frac{k_2}{4} \left(\frac{r_V}{r}\right)^{3/2} \text{diag}\left(-3, 4, \frac{1}{r^2}, \frac{1}{r^2 \sin^2 \theta}\right), \quad (20)$$

and it has therefore the right $-+++$ signature, warranting that the Cauchy problem is well posed and that the scalar field carries positive energy. The large numerical factor $(r_V/r)^{3/2}$ implies that scalar perturbations are weakly coupled to matter; this is a consequence of the Vainshtein mechanism. The different integers entering the diagonal matrix mean that the speed of scalar waves is $c/\sqrt{3}$ in the orthoradial directions, but $2c/\sqrt{3}$ in the radial one. As discussed in [16, 17], this superluminal radial velocity does not violate causality precisely because the Cauchy problem remains well posed simultaneously for all the fields: At any spacetime point, there exists a hypersurface which is spacelike with respect to both $g^{\mu\nu}$ and $\mathcal{G}^{\mu\nu}$, on which one may specify initial data, and such hypersurfaces may be used to foliate the full spacetime.

To conclude, the curved-spacetime Galileon model (1) in an asymptotic Minkowskian Universe is consistent both in the Vainshtein regime ($r \ll r_V$) and at large distances, provided $k_2 > 0$ (and $k_3\alpha > 0$ for the Vainshtein regime to exist). For such an asymptotic Minkowski spacetime, the background scalar field is given by solution (8), (9) or (12) with their *lower* sign.

This conclusion also assumes that α^2/k_2 is not an extremely large dimensionless number, i.e., that matter is not too strongly coupled to the scalar field. If $|\alpha| \gg (|k_3|/M^2 r_S^2)^{1/3}$, then

the $\mathcal{O}(\varphi^4)$ contributions dominate in the effective metric (16) at small enough distances, and its signature becomes $++--$ instead of $-+++$. Scalar perturbations are thus ill behaved, not only because their energy can be negative, but above all because their field equation is not hyperbolic. However, the present study assumed from the beginning that the backreaction of the scalar field on the Schwarzschild metric was negligible, which is obviously not the case in the very strong matter-scalar coupling limit. Therefore, this non-hyperbolic signature does not even prove the inconsistency of the Galileon model (1) in this limit. It just means that we *must* assume as above that α^2/k_2 is at most of order one, otherwise the scalar backreaction on the metric cannot be neglected.

To estimate this backreaction, it suffices to compare the scalar's energy-momentum tensor

$$\begin{aligned} \frac{1}{M_P^2} T_{\mu\nu}(\varphi) = & k_2 \left[2 \partial_\mu \varphi \partial_\nu \varphi - g_{\mu\nu} (\partial_\lambda \varphi)^2 \right] + \frac{k_3}{M^2} \left[2 \square \varphi \partial_\mu \varphi \partial_\nu \varphi \right. \\ & \left. + g_{\mu\nu} \partial_\rho \varphi \nabla^\rho (\partial_\lambda \varphi)^2 - \partial_\mu \varphi \nabla_\nu (\partial_\lambda \varphi)^2 - \partial_\nu \varphi \nabla_\mu (\partial_\lambda \varphi)^2 \right], \end{aligned} \quad (21)$$

with the one of the matter source, or more specifically their spatial integrals $\int_0^r (-T_0^0 + T_i^i) 4\pi r'^2 dr'$, which generate the Newtonian potential $\frac{1}{2}(g_{00} + 1)$ at a distance r from the center of the body. For any r , one then finds that the scalar's contribution is $\mathcal{O}(\alpha^{5/3}(Mr_S)^{2/3})$ smaller than that of matter. Our analysis can thus be trusted only if α is not too large, so that it does not compensate the factor $(Mr_S)^{2/3}$. On the other hand, even if the matter-scalar coupling is of order one (i.e., $\alpha^2/k_2 \sim 1$), then the scalar's backreaction is negligible as soon as $1/M$ is chosen large enough with respect to the Schwarzschild radius of any body. In Galileon models, M is generally assumed to be of the order of the Hubble constant H (and we will actually derive so in Sec. VI below, but while assuming a different asymptotic Universe). Then Mr_S is always an extremely small number, about 10^{-23} for the Sun, 10^{-11} for a galaxy, and still 10^{-8} for a cluster. Therefore, our test scalar field approximation is fully safe if $M \sim H$ and α^2/k_2 is at most of order one.

VI. ASYMPTOTIC DE SITTER SPACETIME

Let us first consider an isotropic and homogeneous Universe, described by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric $ds^2 = -d\tau^2 + a(\tau)^2 (d\rho^2 + \rho^2 d\Omega^2)$, where the cosmological time τ and the comoving radius ρ should not be confused with the time t and radial coordinate r of the Schwarzschild metric (5). Since action (1) does not involve any cosmological constant, the expansion of the Universe will be caused by the Galileon field φ itself. As was shown in [11], the combined Einstein and scalar field equations indeed admit a self-accelerating solution. Let us assume, like in Eq. (6) above, that the scalar field has a linear time dependence, $\varphi = \dot{\varphi}_c \tau + \varphi_0$, i.e., that $\ddot{\varphi} = 0$ (where a dot denotes a derivative with respect to τ). Then the field equations read

$$3H^2 = \frac{\varepsilon}{M_P^2} + k_2 \dot{\varphi}_c^2 - 6H \frac{k_3}{M^2} \dot{\varphi}_c^3, \quad (22a)$$

$$-\dot{H} = \frac{\varepsilon + p}{2M_P^2} + k_2 \dot{\varphi}_c^2 - 3H \frac{k_3}{M^2} \dot{\varphi}_c^3, \quad (22b)$$

$$\frac{\partial_\tau (a^3 J^0)}{a^3} = \alpha(\varepsilon - 3p), \quad (22c)$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, and ε and p denote the energy density and pressure of matter, that we shall neglect in the following. [Actually, Eqs. (22a) and (22c) are valid even if $\ddot{\varphi} \neq 0$.] The only Christoffel contribution to the current (3) comes from $\square\varphi = -3H\dot{\varphi}_c$, and Eq. (22c) reduces to

$$\partial_\tau \left(k_2 a^3 \dot{\varphi}_c - 3H \frac{k_3}{M^2} a^3 \dot{\varphi}_c^2 \right) = 0, \quad (23)$$

which admits the solution $\dot{\varphi}_c = k_2 M^2 / 3H k_3$. [In an expanding Universe, the other solutions, $J^0 = \text{const}/a^3 \rightarrow 0$, tend either towards this one or towards $\dot{\varphi}_c = 0$.] Then Eq. (22b) shows that the scalar-field analogue of $\varepsilon + p$ vanishes, i.e., that it behaves as an effective cosmological constant, and we consistently find that H is also constant. Finally, Eq. (22a) reads $3H^2 = -k_2 \dot{\varphi}_c^2$, implying that k_2 *must* be negative for this self-accelerating solution to exist, and it gives the numerical value

$$H^2 = \frac{M^2}{|k_3|} \left(\frac{|k_2|}{3} \right)^{3/2}. \quad (24)$$

This allows us to rewrite $\dot{\varphi}_c$ in terms of the constants entering action (1). Assuming an expanding Universe (i.e., $H > 0$), we get

$$\dot{\varphi}_c = -\text{sign}(k_3) \left(\frac{|k_2|}{3} \right)^{1/4} \frac{M}{\sqrt{|k_3|}}. \quad (25)$$

Let us now consider a static and spherically symmetric body embedded in such an expanding Universe. As before, we assume that the backreaction of the *local* scalar field on the metric is negligible, but its cosmological time evolution (25) is obviously taken into account since it is responsible for the accelerated expansion. The local scalar-field solution can still be written in the form (8), (9) or (12), at least at small enough distances, and the effective metric in which scalar perturbations propagate is still given by Eq. (16). We saw in Sec. V that the scalar perturbations are ghostlike at large distances if $k_2 < 0$, but this was derived while assuming asymptotic flatness and thereby the Brans-Dicke like behavior (19) of the background scalar field at infinity. Therefore this previous result is no longer valid in the present de Sitter Universe. However, the small-distance physics remains *a priori* unchanged, notably within the Vainshtein radius, and we saw that the signature of the effective metric (16) depended on the sign of $k_3 \square\varphi$. As mentioned below Eq. (12), this sign actually does not depend on k_3 , but it does depend on k_2 , and Eq. (20) confirms that we *a priori* get a ghostlike scalar perturbation if $k_2 < 0$. The stability of the self-accelerating solution seems thus spoiled by the presence of any massive body.

A first way out would be to consider the fine-tuned model such that $\alpha_{\text{eff}} = 0$. Then the scalar field would not be perturbed at all by local matter, there would not exist any Vainshtein regime, and the proof in Ref. [11] that the model is stable would then be valid. This assumption $\alpha_{\text{eff}} = 0$ actually seems more natural in the present self-accelerating Universe, since the condition $\dot{\varphi}_c = (-\alpha/k_3)^{1/2} M$ derived at the end of Sec. III translates as $\alpha = -\text{sign}(k_3) \sqrt{|k_2|/3}$, involving only constant parameters. However, if we take into account the matter sources in Eqs. (22), to derive a more realistic expansion of the Universe, then neither H nor $\dot{\varphi}_c$ will remain constant, and we will eventually reach an epoch for which α is no longer tuned to the right numerical value. Moreover, the self-gravity effects

mentioned in footnote 3 mean that the bare coupling constant α needs to be replaced by a body-dependent product $\alpha(1 - 2s)$, which cannot be fine-tuned to $-\text{sign}(k_3)\sqrt{|k_2|/3}$ for all bodies. Therefore, even for this specific model, one expects that the scalar field will be directly coupled to matter with a nonvanishing α_{eff} , at least at some cosmological epoch, and this seems to generically lead to ghost instabilities because $k_2 < 0$.

In fact, the small-distance effective metric (20) is *not* correct in the present expanding Universe. Indeed, solution (12) does depend on the sign of k_2 , but also on the global \mp sign. We did prove in Sec. V that the lower (+) sign gave the correct solution in an asymptotically Minkowskian Universe, but this is no longer the case in a de Sitter one. This comes from the fact that the FLRW coordinates τ and ρ , useful at cosmologically large distances, do not coincide with the t and r coordinates defining the static and spherically-symmetric local metric (5). In presence of a cosmological constant $\Lambda = 3H^2$, here mimicked by the self-accelerating solution (24), we do know that the exact Schwarzschild-de Sitter solution can be written as Eq. (5) with

$$e^\nu = e^{-\lambda} = 1 - r_S/r - (Hr)^2. \quad (26)$$

This can be matched to an asymptotic FLRW coordinate system by defining

$$\begin{aligned} \tau = t + \frac{1}{2H} \ln [1 - (Hr)^2], & \quad \Leftrightarrow \quad t = \tau - \frac{1}{2H} \ln [1 - (He^{H\tau}\rho)^2], \\ \rho = \frac{e^{-Ht}}{\sqrt{1 - (Hr)^2}} r, & \quad r = e^{H\tau}\rho. \end{aligned} \quad (27)$$

Let us also set

$$B \equiv 1 - \frac{r_S}{e^{H\tau}\rho} - (He^{H\tau}\rho)^2. \quad (28)$$

Then the Schwarzschild-de Sitter metric (5)-(26) becomes

$$ds^2 = -B \left[\frac{d\tau + He^{2H\tau}\rho d\rho}{1 - (He^{H\tau}\rho)^2} \right]^2 + e^{2H\tau} \left[\frac{(d\rho + H\rho d\tau)^2}{B} + \rho^2 d\Omega^2 \right] \quad (29a)$$

$$\begin{aligned} &= - \left[1 - \frac{r_S}{H^2 (e^{H\tau}\rho)^3} + \mathcal{O}\left(\frac{1}{\rho^4}\right) \right] d\tau^2 + \left[\frac{4r_S}{H^3 e^{3H\tau}\rho^4} + \mathcal{O}\left(\frac{1}{\rho^5}\right) \right] d\tau d\rho \\ &\quad + e^{2H\tau} \left\{ \left[1 + \frac{r_S}{H^2 (e^{H\tau}\rho)^3} + \mathcal{O}\left(\frac{1}{\rho^4}\right) \right] d\rho^2 + \rho^2 d\Omega^2 \right\}, \end{aligned} \quad (29b)$$

in which one recognizes the asymptotic de Sitter metric $ds^2 = -d\tau^2 + e^{2H\tau} (d\rho^2 + \rho^2 d\Omega^2)$ up to corrections of order $\mathcal{O}(r_S)$ caused by the local massive body. Note that we merely changed coordinates, and that Eq. (29) still defines the same *static* solution as (5)-(26), in spite of the presence of time-dependent exponentials. In particular, the Schwarzschild radius still corresponds to the constant $e^{H\tau}\rho = r_S$, the factor $e^{H\tau}$ compensating the fact that we now measure lengths with a varying (comoving) ruler. Note also that the spherical body is located at $\rho = 0$ at any time, therefore it is comoving (actually static) in the background de Sitter Universe.

To derive the cosmological solution (24)-(25), we assumed that the scalar field is homogeneous at large distances in de Sitter coordinates, i.e., that it reads $\varphi = \dot{\varphi}_c \tau + \varphi_0$ without any comoving radius (ρ) dependence. This means that in terms of the Schwarzschild coordinates

t and r , Eqs. (27), φ does depend on r . We explicitly get

$$\begin{aligned}\varphi &= \dot{\varphi}_c t + \frac{\dot{\varphi}_c}{2H} \ln [1 - (Hr)^2] + \varphi_0 \\ \Rightarrow \quad \varphi' &= -\dot{\varphi}_c \frac{Hr}{1 - (Hr)^2} = -\frac{k_2 M^2}{3k_3} \frac{r}{1 - (Hr)^2},\end{aligned}\quad (30)$$

H^2 being given by Eq. (24). In other words, the correct solution (8) for the background scalar field φ' , embedded in a self-accelerating Universe, should contain a local r dependence, and with the precise factor $-k_2 M^2/3k_3$ entering Eq. (30). Let us prove that this result is given by the *upper* sign of (8). Indeed, this solution can no longer be expanded as (9) for too large radii r , because expressions (26) for $\nu(r)$ and $\lambda(r)$ must now be used, so that $\nu' e^\nu = -\lambda' e^\nu = r_S/r^2 - 2H^2 r$. We get for such large distances (still assumed smaller than $1/\sqrt{3}H$)

$$\varphi' = -\frac{k_2 M^2}{4k_3} \frac{r}{1 - \frac{3}{2}(Hr)^2} \left[1 \pm \frac{1}{3} \frac{1 - 3(Hr)^2}{1 - (Hr)^2} \right] \times \left[1 + \mathcal{O}\left(\frac{r_S}{r}\right) \right], \quad (31)$$

therefore the upper sign recovers *exactly* the asymptotic behavior derived in Eq. (30) from a matching with the cosmological solution, whereas the lower sign gives this result divided by $[2 - 3(Hr)^2]$, i.e., with not only an erroneous factor $\frac{1}{2}$ by also an incorrect radial dependence.

Solution (8), with its correct upper sign, may now be written for $Hr \ll 1$ as

$$\varphi' = -\frac{k_2 M^2 r}{4k_3} \left(1 + \sqrt{1 - \frac{8}{9} + \frac{4k_3 r_S}{k_2^2 M^2 r^3} \alpha_{\text{eff}}} \right) \times \left[1 + \mathcal{O}\left(\frac{r_S}{r}\right) + \mathcal{O}(H^2 r^2) \right], \quad (32)$$

where the constant $-\frac{8}{9}$ within the square root comes from a compensation between the r_S/r^3 coefficient entering (8) and the large-distance behavior of $\nu' r^2/r_S$, while using the values (24) and (25) for H^2 and $\dot{\varphi}_c$. Note that this solution is different from Eq. (9), which was valid for an asymptotically Minkowskian spacetime. In particular, (32) does not reduce to the Brans-Dicke solution (19a) for $r \gg r_V$ (while still assuming $r \ll 1/H$), but gives

$$\varphi' = -\left(\frac{k_2 M^2 r}{3k_3} + \frac{3 \alpha_{\text{eff}} r_S}{2k_2 r^2} \right) \times \left[1 + \mathcal{O}\left(\frac{r_S}{r}\right) + \mathcal{O}\left(\frac{r_V^6}{r^6}\right) + \mathcal{O}(H^2 r^2) \right], \quad (33)$$

with different sign and numerical coefficient for the usual $\mathcal{O}(1/r^2)$ contribution, but above all involving a much larger $\mathcal{O}(r)$ term, that we already found in Eq. (30) [it is $\mathcal{O}(r^3/r_V^3)$ larger than the $\mathcal{O}(1/r^2)$ contribution].

In the Vainshtein regime ($r_S \ll r \ll r_V$), on the other hand, we still get Eq. (12), but with the crucial information that the upper sign must be chosen:

$$\varphi' = -\frac{k_2 M^2}{4k_3} \sqrt{\frac{r_V^3}{r}} \left[1 + \mathcal{O}\left(\frac{r_S}{r}\right) + \mathcal{O}\left(\frac{r^3}{r_V^3}\right) + \mathcal{O}(H^2 r^2) \right]. \quad (34)$$

This is the opposite expression as the one we used in Sec. V for an asymptotically Minkowskian spacetime, and the effective metric in which scalar perturbations propagate, at small distances, is thus the opposite of (20) [up to negligible corrections of relative order $\mathcal{O}(\sqrt{r_S/r})$ or $\mathcal{O}(\sqrt{r^3/r_V^3})$, due to the nonvanishing time derivative $\dot{\varphi}_c$ we now must take into account when computing (16)]. Since we know that $k_2 < 0$ for the present self-accelerating

Universe to be a solution, this effective metric is therefore again of the correct signature $-+++$, warranting a well-posed Cauchy problem and the absence of ghosts.

The signature of this effective metric must also be studied at large distances $r \gg r_V$. If one neglects all contributions proportional to the Schwarzschild radius r_S of the massive body, $\mathcal{G}^{\mu\nu}$ is most conveniently computed in FLRW coordinates. Then one finds that all contributions to Eq. (16) are of the same order of magnitude $\mathcal{O}(k_2)$, including those of the usually negligible $(\partial\varphi)^4$ terms. The effective metric finally reads $\mathcal{G}^{\mu\nu} = \text{diag}(-2|k_2|, 0, 0, 0) + \mathcal{O}(r_S)$, meaning that the Cauchy problem remains well posed for scalar perturbations, but that their sound velocity vanishes, as was already noticed in [11]. To avoid any instability, the $\mathcal{O}(r_S)$ corrections should thus contribute positively to the effective spatial metric \mathcal{G}^{ij} . We thus need to compute them, while taking into account the zeroth-order terms $\mathcal{O}(r_S^0)$ although we know they will eventually cancel. Indeed some of these zeroth-order terms get multiplied by r_S when solution (8) is expanded to compute (16). Another complication is that $\mathcal{G}^{0r} \neq 0$ in Schwarzschild coordinates (5)-(26), i.e., that $\mathcal{G}^{\mu\nu}$ is not diagonal. However, to determine its signature, it suffices to consider the quadratic form $\mathcal{G}_{\mu\nu}dx^\mu dx^\nu$ (where the effective metric's indices are lowered with $g_{\mu\nu}$, i.e., $\mathcal{G}_{\mu\nu}$ is *not* the inverse of $\mathcal{G}^{\mu\nu}$), and to note that $\mathcal{G}_{00}dt^2 + 2\mathcal{G}_{0r}dtdr + \mathcal{G}_{rr}dr^2 = \mathcal{G}_{00}(dt + \mathcal{G}_{0r}dr/\mathcal{G}_{00})^2 + (\mathcal{G}_{rr} - \mathcal{G}_{0r}^2/\mathcal{G}_{00})dr^2$. In this new frame diagonalizing the effective metric, one finally finds at lowest order $\mathcal{G}_{\mu\nu} \approx \text{diag}(-2|k_2|, 2|k_2|\epsilon, -|k_2|\epsilon r^2, -|k_2|\epsilon r^2 \sin^2\theta)$, where $\epsilon \equiv \frac{3}{4}(r_V/r)^3$, i.e., a signature $-+--$ instead of $-+++$, implying that the Cauchy problem is ill-posed for scalar perturbations propagating in the orthoradial directions (or that they are unstable if one interprets these negative signs as an imaginary sound speed, such instabilities being *large* at moderately large distances of a few r_V). In conclusion, although we found above that the physics of scalar perturbations is consistent in the Vainshtein regime $r \ll r_V$, it is *not* at large distances $r \gg r_V$.

This serious problem can *a priori* have several solutions. The first naive idea would be to choose the theory parameters such that $r_V \gtrsim 1/H$, so that any instability would be hidden behind the cosmological horizon. But since $r_V \propto r_S^{1/3}$, this cannot work for all massive bodies, and this would need anyway a very large matter-scalar coupling constant α , thereby spoiling the hyperbolicity in the Vainshtein regime for the same reasons as at the end of Sec. V (and being inconsistent with our assumption of a negligible backreaction of the local scalar field on the background metric). A much better argument is that the scalar field may never get out of the Vainshtein regime if one takes into account the full matter distribution in the Universe: At any location, there would be a close enough and massive enough body such that $r < r_V$. But the strongest argument is that the vanishing sound speed found above at order $\mathcal{O}(r_S^0)$ is a consequence of the exact self-accelerating solution (24)-(25). As soon as one takes into account matter in the field equations (22), then the sound speed becomes nonvanishing and real, as was shown in [11], and it dominates at large distances over any $\mathcal{O}(r_S)$ correction. For instance, if we assume $\alpha = 0$ like in [11], the presence of a positive matter density ε in Eq. (22a) increases the value of H with respect to the lowest-order expression (24). This implies a modification of Eq. (25) for $\dot{\varphi}_c$, and the matter field equation $\dot{\varepsilon} + 3H\varepsilon = 0$ (assuming a vanishing pressure p) allows us to compute $\ddot{\varphi}_c$. We finally find that the terms proportional to $\square\varphi$ and $(\partial_\lambda\varphi)^4$ in Eq. (16) both give *positive* $\mathcal{O}(\varepsilon)$ contributions to the spatial effective metric \mathcal{G}^{ij} , and we explicitly obtain

$$\mathcal{G}^{ij} = \frac{5}{18} |k_2| g^{ij} \frac{\varepsilon}{M_P^2 H^2} + \mathcal{O}(\varepsilon^2). \quad (35)$$

Surprisingly, the result is fully different if we assume a nonzero bare coupling constant α , because the matter field equation $\dot{\varepsilon} + 3H\varepsilon = \alpha\varepsilon\dot{\varphi}_c$ implies a different time evolution for ε , and the time integration of Eq. (22c) makes α go away. One finds that the lowest-order expressions of H and $\dot{\varphi}_c$ both get multiplied by the same factor $1 - \varepsilon/12M_P^2H^2 + \mathcal{O}(\varepsilon^2)$, and the matter field equation can again be used to compute $\ddot{\varphi}_c$. Now the term proportional to $\square\varphi$ in Eq. (16) happens to give a negative $\mathcal{O}(\varepsilon)$ contribution to \mathcal{G}^{ij} , but it is counterbalanced by the $(\partial_\lambda\varphi)^4$ and $\nabla^\mu\partial^\nu\varphi$ terms of (16). We finally get

$$\mathcal{G}^{ij} = \frac{1}{18} |k_2| g^{ij} \frac{\varepsilon}{M_P^2 H^2} \left(1 + \text{sign}(k_3) \alpha \sqrt{\frac{3}{|k_2|}} \right) + \mathcal{O}(\varepsilon^2), \quad (36)$$

which is positive definite if $3\alpha^2/|k_2| < 1$ (and for even larger $|\alpha|$ if $k_3\alpha > 0$). The reason why (36) does not tend to (35) when $\alpha \rightarrow 0$ is because they correspond to different cosmological initial conditions. In any case, their positivity shows that the ill-posed Cauchy problem found above was the consequence of an oversimplified cosmological background, and the signature of the effective metric at large distances is in fact $-+++$, as needed.

To conclude, the curved-spacetime Galileon model (1) with $k_2 < 0$ admits a self-accelerating cosmological solution, and a spherical body embedded in this Universe generates a scalar field given by the *upper* sign of Eq. (8), i.e., by (32), (33) or (34) depending on the distance to this body — the Vainshtein regime (34) existing only if $k_3\alpha_{\text{eff}} > 0$. Perturbations around this solution carry positive energy and have a well-posed Cauchy problem in the Vainshtein regime $r \ll r_V$. Farther away, one must take into account the matter content of the Universe to derive the cosmological expansion, and the effective metric in which scalar perturbations propagate then remains of the correct hyperbolic signature $-+++$.

Let us finally mention in which conditions the local scalar field's backreaction on the metric is indeed negligible, as was assumed to draw the above conclusions. First, this is always the case if α_{eff} is small enough, i.e., if one tunes the bare matter-scalar coupling constant to $\alpha \approx -\text{sign}(k_3)\sqrt{|k_2|/3}$, so that it almost compensates the cosmologically induced coupling $k_3\dot{\varphi}_c^2/M^2$ entering Eq. (10). Indeed, the spatial integral of Eq. (21) over a sphere of radius $r < 1/H$ is at most of order $\mathcal{O}(\alpha_{\text{eff}} r_S)$, negligible with respect to the effect of the material body of mass $\frac{1}{2}r_S$. However, for a random bare coupling constant such that $\alpha^2/|k_2| \lesssim 1$ (a small value being also allowed), Eqs. (10) and (25) imply that $\alpha_{\text{eff}}^2/|k_2|$ is generically of the order of unity, therefore one actually expects large deviations from our results above. But even in such a case, the scalar field's backreaction is still negligible in the Vainshtein regime $r \ll r_V$, because it is of order $\mathcal{O}(\alpha_{\text{eff}} r_S (r/r_V)^{3/2})$. Therefore, our previous conclusion that the Galileon model (1) is stable in the Vainshtein regime remains valid even for a matter-scalar coupling of order one. On the other hand, for such a $\mathcal{O}(1)$ coupling, our test scalar field approximation cannot be trusted at distances $r \gtrsim r_V$, and further work is needed to actually prove stability [18]. Finally, for cosmologically large distances $r \sim 1/H \gg r_V$, both the effects of the massive body and the locally generated scalar field are negligible, and we asymptotically reach an effective metric $\mathcal{G}^{\mu\nu} \approx |k_2| \text{diag}(-2, \mathcal{O}(1)\varepsilon g^{ij}/M_P^2 H^2)$, with the correct hyperbolic signature $-+++$.

VII. SCALAR WAVES WITHIN MATTER

Although we have shown in Secs. V and VI that the field equations for scalar perturbations are hyperbolic in the Vainshtein regime, and thereby that there is no ghost instability at

small enough distances from a massive body, the growth of such perturbations *within* the body deserves a more careful study. First of all, the vacuum solution (8) for the scalar field is no longer valid inside matter, therefore the hyperbolicity of the effective metric (16) needs to be checked. To simplify, let us consider a spherical body of constant density. As derived in page 331 of Ref. [19], the interior solution for the metric (5) (neglecting cosmological corrections of order $H^2 r^2$) reads

$$e^{\nu(r)} = \frac{1}{4} \left[3 \left(1 - \frac{r_S}{r_*} \right)^{1/2} - \left(1 - \frac{r_S r^2}{r_*^3} \right)^{1/2} \right]^2 = 1 - \left(3 - \frac{r^2}{r_*^2} \right) \frac{r_S}{2r_*} + \mathcal{O}(r_S^2), \quad (37a)$$

$$e^{\lambda(r)} = \left(1 - \frac{r_S r^2}{r_*^3} \right)^{-1} = 1 + \frac{r_S r^2}{r_*^3} + \mathcal{O}(r_S^2), \quad (37b)$$

where r_* denotes the body's radius, not to be confused with its Schwarzschild radius r_S . The background scalar field equation (2) can then be solved as in Sec. III, with the difference that only the mass interior to the sphere of radius r , $m(r) = (r_S/2G)(r/r_*)^3$, is a source for $\varphi'(r)$. We get

$$\varphi'_{\text{interior}} = -\frac{k_2 M^2 r}{4k_3} \left(1 \pm \sqrt{1 - \frac{8H^2 k_3^2}{k_2^2 M^4} \dot{\varphi}_c^2 + \frac{4k_3 r_S}{k_2^2 M^2 r_*^3} \alpha_{\text{eff}}} \right) + \mathcal{O}(r_S^2) + \mathcal{O}(H^2 r^2), \quad (38)$$

where either $H = 0$ in asymptotic Minkowski spacetime, or H and $\dot{\varphi}_c$ are given by Eqs. (24) and (25) in asymptotic de Sitter spacetime (yielding the constant $-\frac{8}{9}$ we already encountered in Eq. (32) for the exterior solution). The only difference with respect to the exterior solution (9) or (32), at this lowest post-Newtonian order, is that α_{eff} is multiplied by the constant $1/r_*^3$ instead of $1/r^3$. In the Vainshtein regime $r \leq r_* \ll r_V$, we have thus

$$\varphi'_{\text{interior}} = \mp \frac{k_2 M^2}{4k_3} \left(\frac{r_V}{r_*} \right)^{3/2} r \left[1 + \mathcal{O}\left(\frac{r_S}{r_*}\right) + \mathcal{O}\left(\frac{r_*^3}{r_V^3}\right) + \mathcal{O}(H^2 r_*^2) \right], \quad (39)$$

instead of Eq. (12), where the lower sign (+) corresponds to asymptotic Minkowski spacetime, and the upper one (−) to asymptotic de Sitter spacetime. The various contributions to the effective metric (16) can now be computed within the body, and one finds at lowest order (in both asymptotic Minkowski and de Sitter cases)

$$\mathcal{G}_{\text{interior}}^{\mu\nu} \approx |k_2| \left(\frac{r_V}{r_*} \right)^{3/2} \text{diag} \left(-\frac{3}{2}, 1, \frac{1}{r^2}, \frac{1}{r^2 \sin^2 \theta} \right), \quad (40)$$

instead of (20) [or the opposite of of (20) in the asymptotic de Sitter case, for which $-k_2 = |k_2|$]. Note that this effective metric is *discontinuous* at the surface of the body, because it involves φ'' , which is discontinuous when one suddenly passes from empty space to a finite matter density. But of course, everything becomes continuous when one considers a smooth matter profile. Note also that the sound speed, $\sqrt{2/3}c$, is now isotropic, and differs from its value outside matter (that we found in Sec. V to be $2c/\sqrt{3}$ in the radial direction and $c/\sqrt{3}$ in the orthoradial ones). This can be interpreted as different refractive indices of matter and vacuum for scalar waves. But the crucial information brought by Eq. (40) is that the effective metric remains of hyperbolic signature $-+++$ within matter, therefore no ghost instability develops even in the interior of a body.

On the other hand, the Ricci tensor entering Eq. (4) no longer vanishes within matter, and this causes extra couplings of the perturbations to the background fields, whose effects could *a priori* lead to other types of instabilities. This Ricci tensor is actually generated both by matter and by the background scalar field itself. The scalar-scalar self-interactions are responsible for the $\mathcal{O}(k_3^2)$ terms in the effective metric (16), as was first shown in [11]. Not only we already took them into account to derive (40), but we also saw in Secs. V and VI that they are negligible with respect to the $\mathcal{O}(k_3)$ terms, provided $\alpha^2/|k_2|$ is at most of order 1 (i.e., that we are not considering an extremely large matter-scalar coupling constant). We can thus focus on the Ricci tensor generated by matter, $R_{\text{matter}}^{\mu\nu} = \frac{3}{2}(r_S/r_*^3)g^{\mu\nu} + \mathcal{O}(r_S^2)$, which multiplies *first* derivatives of the scalar field in Eq. (4), and thereby does not change its kinetic term (second derivatives). Nevertheless, its presence means that within matter, scalar perturbations acquire a direct derivative coupling to the background scalar field. The first-order expansion of (4) reads

$$\square\varphi\square\pi - (\nabla^\mu\partial^\nu\varphi)(\nabla_\mu\partial_\nu\pi) - R_{\text{matter}}^{\mu\nu}\varphi_{,\mu}\pi_{,\nu} = 0, \quad (41)$$

where we have neglected the $\mathcal{O}(k_3^2)$ terms coming from the diagonalization (14)–(16) of the kinetic terms, as well as the $(k_2 M^2/2k_3)\square\pi$ linear term because we assume the star is in the Vainshtein regime ($r_* \ll r_V$). At lowest post-Newtonian order, the radial contribution $-R_{\text{matter}}^{rr}\varphi'\pi'$ can also be neglected, because both $R_{\text{matter}}^{rr} = \mathcal{O}(r_S)$ and $\varphi' = \mathcal{O}(\sqrt{r_S})$ tend to 0 with r_S . But the temporal contribution $-R_{\text{matter}}^{00}\dot{\varphi}_c\dot{\pi}$ is crucial, as it corresponds to a damping or antidamping term depending on its sign. Let us thus consider the asymptotic de Sitter case of Sec. VI, where $\dot{\varphi}_c$ is given by Eq. (25). The main contributions to Eq. (41) then read

$$\begin{aligned} \nabla_\mu(\mathcal{G}_{\text{interior}}^{\mu\nu}\partial_\nu\pi) - \frac{2k_3}{M^2}R_{\text{matter}}^{00}\dot{\varphi}_c\dot{\pi} &= 0 \\ \Leftrightarrow -\ddot{\pi} + \frac{2}{3}\Delta\pi - \left(\frac{|k_2|}{3\alpha_{\text{eff}}^2}\right)^{1/4}\left(\frac{r_S}{r_*^3}\right)^{1/2}\dot{\pi} &= 0. \end{aligned} \quad (42)$$

The coefficient of the $\dot{\pi}$ term is generically of order $\sqrt{r_S/r_*^3}$, notably when the bare matter-scalar coupling constant α vanishes or is negligibly small. Indeed, Eqs. (10) and (25) then give $\alpha_{\text{eff}}^2 \approx |k_2|/3$. Note that this coefficient can never be small with respect to $\sqrt{r_S/r_*^3}$, because we assume that $\alpha^2/|k_2|$ is at most of order 1, therefore $\alpha_{\text{eff}}^2/|k_2|$ is never large either. On the other hand, it is possible to choose the bare α so that α_{eff} is small (see Sec. VI), and the $\dot{\pi}$ term of Eq. (42) may thus be multiplied by a large number in some specific situations. Therefore, the extra coupling of the perturbation π to the cosmologically-imposed $\dot{\varphi}_c$ *within* matter can change drastically its behavior. The good news is that its sign implies this is a friction term, and therefore that no scalar instability occurs within the material body. The plane-wave solutions of Eq. (42) read indeed (in Cartesian coordinates)

$$\pi \propto e^{-t/T} \sin(\omega t - \mathbf{k} \cdot \mathbf{x} + \text{const}), \quad (43)$$

with a dispersion relation

$$\frac{2}{3}\mathbf{k}^2 = \omega^2 + \frac{1}{T^2}, \quad (44)$$

and a decay time

$$T \equiv 2 \left(\frac{3\alpha_{\text{eff}}^2}{|k_2|} \right)^{1/4} \left(\frac{r_*^3}{r_S} \right)^{1/2}. \quad (45)$$

For the Sun, $2\sqrt{r_*^3/r_S}$ is about one hour, therefore this damping is always quite efficient. But when α_{eff} happens to be small, because of a balance between the bare matter-scalar coupling constant α the cosmologically-induced one $k_3\dot{\varphi}_c^2/M^2$ in Eq. (10), scalar perturbations are suppressed even faster. For $\omega = 0$ and $\frac{2}{3}\mathbf{k}^2 \leq 1/T^2$, there exist other solutions of the form $\pi \propto e^{-t/T_{\pm}} \sin(\mathbf{k} \cdot \mathbf{x} + \text{const})$, where $1/T_{\pm} \equiv 1/T \pm \sqrt{1/T^2 - 2\mathbf{k}^2/3}$ is always positive, therefore such perturbations are damped too. The upper sign gives an even faster decay than for the plane waves (43), whereas the lower sign corresponds to a slower damping. The limiting case of a homogeneous perturbation ($\mathbf{k} = \mathbf{0}$) gives either a fast damping $\pi \propto e^{-2t/T}$, or an actual constant which can be reabsorbed in the definition of the background φ_0 .

In conclusion, the physics of scalar perturbations is fully safe in the interior of a spherical body in the Vainshtein regime. Not only the Cauchy problem is well posed and the kinetic energy carried by scalar perturbations is positive, but even the subtle derivative coupling caused by the nonvanishing Ricci tensor in Eq. (41) efficiently *damps* these perturbations. Note that this local friction within material bodies is paradoxically caused by the slow cosmological time evolution ($\dot{\varphi}_c \neq 0$) of the background scalar field. The large-distance behavior of the Universe is thus significantly influencing local physics.

VIII. CONCLUSIONS

In this paper we studied in detail spherically symmetric solutions for the cubic covariant Galileon model (1) in presence of a matter source. An important ingredient of our study is that we take into account the time variation of the scalar field, induced by the cosmological evolution. Although cosmological time evolution is tiny (normally of order of the Hubble rate, $\dot{\varphi}_c \sim H$), nevertheless its effect on the behavior of the scalar is crucial and cannot be disregarded.

We exhibited, in particular, an *induced* matter-scalar coupling for $\dot{\varphi}_c \neq 0$, Eq. (10). This effect arises because of the kinetic mixing of the Galileon and the graviton, which results in the coupling of the derivatives of the scalar field to the curvature, in the Galileon field equation. For $\dot{\varphi}_c \neq 0$, this term plays the role of a matter-scalar coupling, hence it is *induced* by the cosmological evolution of φ . It is important to note that this coupling is naturally of order of unity, therefore the Galileon effectively couples to matter sources even if the bare scalar-matter coupling is absent, $\alpha = 0$.

We also found that the local solution for the Galileon field crucially depends on the asymptotic conditions at large distances. Indeed, to find time-dependent solutions in presence of massive sources, we applied the ansatz (6), which enables to separate the time and radial variables. The radial-dependent part can then be found exactly, in the test scalar field approximation, for an arbitrary static background metric, Eq. (8). [We checked *a posteriori* the conditions for backreaction to be negligible.] The full solution for φ' , however, consists of two different branches. The solution with the *lower* sign in (8) corresponds to the asymptotically Minkowski spacetime, with no time evolution of φ . The other case we studied in detail is the asymptotically de Sitter Universe, whose self-acceleration is provided by a time evolution of the Galileon itself. In this case, the cosmological asymptotic behavior dictates to choose the other branch of the solution, namely, Eq. (8) with the *upper* sign. It turns out that the stability of the solutions drastically depends of the choice of the branch.

In order to investigate stability of the found solutions, notably the well-posedness of the Cauchy problem and the positivity of energy, we identified the actual spin-0 and spin-

2 degrees of freedom by diagonalizing their kinetic terms in Sec. IV. Using the results of Sec. IV, we found in Sec. V that the background scalar field given by solution (8), (9) or (12) with their *lower* sign (corresponding to the asymptotically Minkowski spacetime), is consistent both in the Vainshtein regime ($r \ll r_V$) and at large distances, provided $k_2 > 0$ and $k_3\alpha \geq 0$.

The asymptotically de Sitter case has been treated separately in Sec. VI. It should be noted that the self-accelerating de Sitter stage is only possible for the choice $k_2 < 0$ in action (1). Therefore naively one would expect appearance of a ghost instability at least in the Vainshtein regime, see e.g. Eq. (20), where a negative k_2 means ghost scalar perturbation. One should also have in mind that the existence of the Vainshtein regime is generic, even with no direct scalar-matter coupling, because of the *induced* coupling for time-dependent solutions. It turns out, however, that the choice of the correct branch, corresponding to the asymptotically de Sitter solution, namely, Eq. (8) with the *upper* sign (in contrast to the asymptotically Minkowski where the *lower* sign is taken), changes drastically the behavior of perturbations. Indeed, we found that the solution is stable for the Vainshtein regime, contrary to naive expectations.

The large distance behavior, $r \gg r_V$, for the asymptotic de Sitter is more subtle. The pure cosmological perturbations of the spin-0 degree of freedom have positive-energy dust-like behavior (i.e., with vanishing sound speed). Therefore any small deviation can potentially create a change in signature for perturbations. In fact, the deviations caused by a local material body do not yield a correct signature: The field equations for perturbations become elliptic in the angular directions. However, this problem exists only in the pure de Sitter universe filled with the Galileon field with no external matter. In a more realistic scenario, when a small amount of normal matter is added (such that the universe is almost de Sitter), the effective metric (16) always has the correct signature.

Another interesting effect, associated with the cosmological time evolution of the Galileon, is the appearance of a significant friction term in the equation of motion. Because of the commutation of covariant derivatives, the Galileon field equations involve couplings of the scalar field to the curvature tensor. In the cubic model (1) studied in this paper, this generates *within* matter an extra interaction of scalar perturbations with their background. We showed that it causes an efficient damping of these perturbations, a local effect paradoxically caused by the cosmological time evolution.

To summarize, we studied spherically symmetric solutions of the cubic covariant Galileon model in presence of a matter source and a cosmological evolution. We found that the time evolution of the Galileon, even as small as a cosmological one, leads to considerable effects: in particular, the appearance of an *induced* coupling, which is generically of the order of unity, so that the Galileon becomes effectively coupled to local matter sources even if the bare coupling is zero; and the emergence of a friction term, which effectively *damps* perturbations within matter sources. The detailed analysis of perturbations showed that the Galileon model is well behaved in asymptotically Minkowski space as well as in the asymptotically de Sitter, provided that the correct branch of the solution is chosen.

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- [1] G. W. Horndeski, *Int. J. Theor. Phys.* **10**, 363 (1974).
 - [2] D. B. Fairlie, J. Govaerts, and A. Morozov, *Nucl. Phys. B* **373**, 214 (1992) [hep-th/9110022]; D. B. Fairlie and J. Govaerts, *J. Math. Phys.* **33**, 3543 (1992) [hep-th/9204074]; *Phys. Lett. B* **281**, 49 (1992) [hep-th/9202056].
 - [3] G. R. Dvali, G. Gabadadze, and M. Porrati, *Phys. Lett. B* **485**, 208 (2000) [arXiv:hep-th/0005016].
 - [4] A. Nicolis, R. Rattazzi, and E. Trincherini, *Phys. Rev. D* **79**, 064036 (2009) [arXiv:0811.2197 [hep-th]].
 - [5] C. Deffayet, G. Esposito-Farèse, and A. Vikman, *Phys. Rev. D* **79**, 084003 (2009) [arXiv:0901.1314 [hep-th]].
 - [6] C. Deffayet, S. Deser, and G. Esposito-Farèse, *Phys. Rev. D* **80**, 064015 (2009) [arXiv:0906.1967 [gr-qc]]; *Phys. Rev. D* **82**, 061501(R) (2010) [arXiv:1007.5278 [gr-qc]].
 - [7] C. Deffayet, X. Gao, D. A. Steer, and G. Zahariade, *Phys. Rev. D* **84**, 064039 (2011) [arXiv:1103.3260 [hep-th]].
 - [8] E. Babichev, C. Deffayet, and R. Ziour, *Int. J. Mod. Phys. D* **18**, 2147 (2009) [arXiv:0905.2943 [hep-th]].
 - [9] C. Burrage and D. Seery, *JCAP* **1008**, 011 (2010) [arXiv:1005.1927 [astro-ph.CO]]; P. Brax, C. Burrage, and A. -C. Davis, *JCAP* **1109**, 020 (2011) [arXiv:1106.1573 [hep-ph]]; E. Babichev, C. Deffayet, and G. Esposito-Farèse, *Phys. Rev. D* **84**, 061502(R) (2011) [arXiv:1106.2538 [gr-qc]]; N. Kaloper, A. Padilla, and N. Tanahashi, *JHEP* **1110**, 148 (2011) [arXiv:1106.4827 [hep-th]]; A. De Felice, R. Kase, and S. Tsujikawa, *Phys. Rev. D* **85**, 044059 (2012) [arXiv:1111.5090 [gr-qc]]; E. Bellini, N. Bartolo, and S. Matarrese, *JCAP* **1206**, 019 (2012) [arXiv:1202.2712 [astro-ph.CO]]; S. Deser and J. Franklin, *Phys. Rev. D* **86**, 047701 (2012) [arXiv:1206.3217 [gr-qc]]; C. de Rham, A. J. Tolley, and D. H. Wesley, arXiv:1208.0580 [gr-qc]; T. Hiramatsu, W. Hu, K. Koyama, and F. Schmidt, arXiv:1209.3364 [hep-th]; Y. -Z. Chu and M. Trodden, arXiv:1210.6651 [astro-ph.CO]; A. V. Belikov and W. Hu, arXiv:1212.0831 [gr-qc].
 - [10] E. Babichev, C. Deffayet, and G. Esposito-Farèse, *Phys. Rev. Lett.* **107**, 251102 (2011) [arXiv:1107.1569 [gr-qc]].
 - [11] C. Deffayet, O. Pujolas, I. Sawicki, and A. Vikman, *JCAP* **1010**, 026 (2010) [arXiv:1008.0048 [hep-th]].
 - [12] P. Jordan, *Nature (London)* **164**, 637 (1949); *Schwerkraft und Weltall* (Vieweg, Braunschweig, 1955); *Z. Phys.* **157**, 112 (1959); M. Fierz, *Helv. Phys. Acta* **29**, 128 (1956); C. Brans and R. H. Dicke, *Phys. Rev.* **124**, 925 (1961).
 - [13] C. M. Will, *Theory and experiment in gravitational physics* (Cambridge University Press, Cambridge, 1993).
 - [14] T. Damour and G. Esposito-Farèse, *Class. Quant. Grav.* **9**, 2093 (1992); *Phys. Rev. D* **53**, 5541 (1996), [gr-qc/9506063].
 - [15] E. Babichev, *Phys. Rev. D* **83**, 024008 (2011) [arXiv:1009.2921 [hep-th]].
 - [16] J.-P. Bruneton, *Phys. Rev. D* **75**, 085013 (2007) [gr-qc/0607055]; J.-P. Bruneton and

- G. Esposito-Farèse, Phys. Rev. D **76**, 124012 (2007) [arXiv:0705.4043 [gr-qc]].
- [17] E. Babichev, V. Mukhanov, and A. Vikman, JHEP **0802**, 101 (2008) [arXiv:0708.0561 [hep-th]].
- [18] E. Babichev *et al.*, in progress.
- [19] S. Weinberg, *Gravitation and cosmology* (John Wiley and Sons, New-York, 1972).